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ABSTRACT

The iterative interlacing error diffusion (IIED) technique consists of the combination of the error diffusion (ED) and the modified iterative interlacing (IIT) techniques to synthesize computer-generated holograms. The IIED technique leads to a fairly dramatic improvement in the quality of reconstructed images provided that the two constant parameters involved in iterations are properly chosen.

I. INTRODUCTION

The error diffusion(ED) method originally introduced by Floyd and Steinberg[1] as a halftoning technique has been successfully applied to a number of applications. Hauck and Bryngdahl[2] were the first to realize the applicability of ED to computer holography. Compared with iterative approaches such as the direct-binary-search3 (DBS) and projections onto constraint sets[4] (POCS), various types[5-7] of ED algorithms have shared the sarne major advantage of faster computation. On the other hand, for continuous or finely quantized amplitude or phase holograms, better performance is generally obtained with iterative procedures.

A stagnation problem[8] restricts the application of the POCS method in designing holograms with quantized amplitudes or phases. Wyrowski[7] attempted to solve this problem by stepwise introduction of quantization constraints. The total cornputation time with this modification is increased ,say, approximately ten times if a ten-step quantization per iteration is used. A recent approach called the iterative interlacing technique[9] (IIT) avoids the stagnation problem and gives excellent results without increasing computation time.

In this article, a new approach which combines the iterative interlacing technique and error diffusion is discussed to permit the preservation of the advantages of both methods while reducing their shortcomings and avoiding the stagnation problem. The resulting technique is referred to as the iterative interlacing error diffusion (IIED) technique.

In the following sections, the holograms to be discussed are Fourier holograms. In Fourier transform holography, the front and the back focal planes of a lens are used as the hologriim and the image planes. Then, the transformation between the two planes is essentially the Fourier transform, which is approximated in numerical computations by the discrete Fourier transform (DFT). Hence,we will describe the algorithms in the following sections in terms of discrete-space signals and discrete-space transforms.

The paper consists of six sections. Section 2 discusses the IIT technique and a particular modification of it, which is used in the IIED technique in Section 4. Section 3 briefly describes ED algorithms. The IIED technique is introduced in Section 4. The computer experiments showing the effectiveness of the IIED technique as compared to the IIT and the ED techniques are presented in Section 5. Conclusions are reached in Section 6.

II. ITERATIVE INTERLACING TECHNIQUE

The IIT approach[9], which has been conceptually described as hierarchically designing and interlacing a number of holograms to add up coherently to a single desired reconstruction, proves to be very effective in reducing reconstruction error and speeding up the convergence time. Each subsequent hologram is designed to reduce the reconstruction error obtained previously.

In this article, we will consider interlacing two holograms with the geometry shown in Fig.1. The first hologram is designed for the odd-numbered rows of the total hologram and the second hologram is designed for the rest. Let X(n1,n2) be the desired object image, and $Xrec_1(n1,n2)$ be the reconstructed image generated by the first hologram,.The resulting error irnage can be written as

$$E_1(n1,n2) = X(n1,n2) - \lambda_1 \, Xrec_1(n1,n2) \qquad (1)$$

where λ_1 is a scaling factor. The technique of computation of λ_1 is discussed in Ref. [9]. The second hologram is then designed to reconstruct $E_1(n1,n2)/\lambda_1$. If the second hologram were perfect, the sum of the two reconstructions would be equal to $X(n1,n2)/\lambda_1$, which differs from the desired image only by a scaling factor. This being not case,the total reconstruction yields an error image given by

$$E_2(n1,n2) = X(n1,n2) - \lambda_2 (Xrec_1(n1,n2) + Xrec_2(n1,n2)) \tag{2}$$

where $Xrec_2(n1,n2)$ is the reconstructed image generated by the second hologram, and λ_2 is the scaling factor associated with the total reconstruction after the second hologram is designed. This completes the first sweep of designing the two holograms. In the second sweep, $E_2(n1,n2)$ is circulated back to the first hologram to reduce the reconstruction error further. In the previous paper[9], this was done by modifying the desired image for the first hologram as

$$X_{2,1}(n1,n2) = Xrec_1(n1,n2) + E_2(n1,n2)/\lambda_2 \tag{3}$$

and the desired image for the second hologram as

$$X_{2,2}(n1,n2) = Xrec_2(n1,n2) + E_{2,1}(n1,n2)/\lambda_{2,1} \tag{4}$$

where $E_{2,1}(n1,n2)$ and $\lambda_{2,1}$ are the new error image and the scaling factor for the first hologram,respectively. Similar procedures of modifying the desired images are then applied in the subsequent designs. This process is continued for a number of sweeps until convergence or some error criterion is met. The implementations together with the DBS and the POCS algorithms have been shown[9] to decrease the reconstruction error steadily.

A modification of the IIT technique to obtain further improvement consists of updating Eqs. (3) and (4) as follows:

$$X_{i,1}(n1,n2) = \beta\, X_{i-1,1}(n1,n2) + E_{i-1,2}(n1,n2)/\lambda_{i-1,2} \tag{5}$$

$$X_{i,2}(n1,n2) = \beta\, X_{i-1,2}(n1,n2) + E_{i,1}(n1,n2)/\lambda_{i,1} \tag{6}$$

where $X_{i,1}(n1,n2)$ and $X_{i-1,1}(n1,n2)$ are the desired images for the first hologram in the ith and (i-1)th sweeps, respectively; $E_{i-1,2}(n1,n2)$, $\lambda_{i-1,2}$ are the total reconstruction error and the normalization constant after the design of the second hologram in the (i-1)th sweep,respectively; β is a weighting factor between the previous inputs and the desired correction terms.The terms $X_{i,2}(n1,n2)$, $E_{i,1}(n1,n2)$, and $\lambda_{i,1}$ are similarly defined. It is the modified IIT technique which is mostly used in Section 4. Similar procedures of modifying the input image to improve the performance and speed of convergence of iterative procedures were used by Fienup[10] for the problem of recovering phase from intensity measurements, following the argument that a small change of the desired image results in a change of the reconstructed image in the same general direction as the change of the desired image.

III. ERROR DIFFUSION ALGORITHMS

In general, the ED algorithms can be applied to the design of both real valued[2] and complex-valued[7] computer-generated holograms. In this article, only real binary holograms will be considered. Assuming the values of the pixels have been normalized between 0 and 1, Fig. 2 illustrates an example of the sequential binarization process. The first pixel H_1 is clipped by the hard-limiter having threshold T as shown in Fig. 3, yielding a binary value B_1 (0 or 1) and an error E_1 defined as

$$E_1 = H_1 - B_1 \tag{7}$$

This error is then added the next pixel. The same procedure is repeated with all the following pixels.

The modified versions of the ED algorithm use the same scheme, and differ only in their ways of distributing error. By properly choosing the direction of the error diffusion, one can separate the noise clouds from the desired image field reasonably well.

M. Broja, ect.[11] showed that the stability of ED methods depends on the choice of the weighting factors. When each generated error is diffused only to the next following pixel with a diffusion coefficient w, the process can be described as

$$\tilde{H}_m = H_m + \omega\, E_{m-1} \tag{8}$$

$$B_m = f(\tilde{H}_m - T) \tag{9}$$

$$E_m = \tilde{H}_m - B_m \tag{10}$$

where $0 \blacksquare H_m \le 1$; $0 \le T \blacksquare 1$; f(.) is the hard-limiter, and T is a constant threshold. The error E_m associated with the mth pixel can then be rewritten as

$$E_m = \omega E_{m-1} + (H_m - B_m) \tag{11}$$

In the case of instability, since $-1 <= (H_m - B_m) <= 1$, we can assume that the absolute value of E_m becomes much larger than 1, and obtain

$$E_m = \omega E_{m-1} = \omega^{m-1} E_1 \tag{12}$$

Depending on the sign of w , there are two different types of instability. A positive w (w > 1) leads to a monotonously increasing or decreasing error which depends on the sign of E_1 while a negative ω (w < -1) leads to an increasing, oscillating error. In general, $|\omega| < 1$ is sufficient to limit the value of E_m. For this reason, ω less than 1 is used in the computer experiments described in Sec. V.

IV. ITERATIVE INTERLACING ERROR DIFFUSION

By combining the iterative interlacing and the error diffusion techniques, we can achieve improvement in performance with reasonable computation time. A one-dimensional version of the ED algorithm as described above is adopted here to code the hologram, due to its strong capability of reducing reconstruction error[6] and its compatibility to IIT.

Real-valued holograms are realized by using Hermitian symmetry. We divide the image plane into four quadrants as shown in Fig. 4. Two quadrants contain the desired image and its Hermitian image surrounded with zeros, and the other two quadrants contain only zeros. This is required for two reasons. First, in order to satisfy the sampling theorem and to avoid overlapping the object with the unwanted terms, especially a strong DC peak ,its twin image and noise from higher diffraction orders , error diffusion techniques require the object to be surrounded by zeros in a larger data field, and to be shifted from the center by a large enough offset.. Secondly, the interlacing technique needs the two quadrants which do not contain the object images to be zero. This can be explained as follows:

Let a hologram $H(k1,k2)$ have a corresponding Fourier domain image $X(n1,n2)$. Following the interlacing concept described in Sec. 2, we define an odd-row hologram $H'(k1,k2)$ of $H(k1,k2)$ as

$$H'(k1,k2) = H(k1,k2) \quad \text{for } k1 \text{ odd} \tag{13}$$

$$= 0 \quad \text{otherwise}$$

which can be rewritten as

$$H'(k1,k2) = 0.5((-1)^{k1} + 1)H(k1,k2) \tag{14}$$

Its corresponding Fourier domain image $X'(n1,n2)$ can then be expressed as

$$X'(n1,n2) = 0.5[X(n1,n2) + X(n1+N/2,n2)] \tag{15}$$

i.e., the Fourier domain image corresponding to $H'(k1,k2)$ is the sum of $X(n1,n2)$ and its shifted version. With zero quadrants, this overlapping will not cause any distortion in the image quadrant and its Hermitian quadrant except for a scaling factor. Similar results can be obtained for even-row holograms.

In particular, when the desired image occupies only a portion of the image quadrant, as shown in Fig.5(a) surrounded with zeros, the corresponding odd-row and even-row holograms have an undisturbed image in the same position, as shown in Fig.5(b) and Fig.5(c). In all three cases of Fig.5, despite half the object fields are different, the amplitude freedom of the background surrounding object fields initialized with zeros is the same. Hence, it is reasonable to believe that, in terms of reconstruction error associated with the desired image, similar results from ED might be obtained for all three cases. This implies that holograms corresponding to the first case generated by ED may contain a high degree of redundancy. In other words, by using odd rows(or even rows) alone, one can have a comparable result to the one generated by designing all rows togetlher. Then, one can use not yet designed rows as extra parameters to reduce the reconstruction error further. By this reasoning, we expect that the IIED technique should outperform the ED technique alone.

In the following procedure of the IIED technique, the index i is used to specify the number of sweeps, and the index j equal to 1 or 2 is used to distinguish the two holograms. The algorithm starts with initializing the desired image $X(n1,n2)$, sets $i = 1$, $j = 1$ and computes the following:

1. In the ith sweep,the quadrant image $X_{i,j}(n1,n2)$, which the jth hologram is designed to reconstruct, is updated according to

$$\begin{aligned} X_{i,j}(n1,n2) &= X(n1,n2) && i = 1, j = 1 \\ &= X(n1,n2)/\lambda_{i,j} - Xrec1(n1,n2) && i = 1, j = 2 \\ &= \beta X_{i-1,j}(n1,n2) + E_{i-1,2}(n1,n2)/\lambda_{i-1,2} && i > 1, j = 1 \\ &= \beta X_{i-1,j}(n1,n2) + E_{i-1,1}(n1,n2)/\lambda_{i-1,1} && i > 1, j = 2 \end{aligned} \tag{16}$$

2. Prepare a four-quadrant image as shown in Fig. 4.

3. Compute the discrete Fourier transform (DFT) of the four-quadrant image and denote the resulting hologram image $H_j(k1,k2)$.

4. If j = 1, define

$$\begin{aligned} H'_j(k1,k2) &= H_j(k1,k2) && \text{for } k1 \text{ even} \\ &= 0 && \text{otherwise} \end{aligned} \tag{17}$$

if j = 2, define

$$\begin{aligned} H'_j(k1,k2) &= H_j(k1,k2) && \text{for } k1 \text{ odd} \\ &= 0 && \text{otherwise} \end{aligned} \tag{18}$$

5. Shift and normalize $H'_j(k1,k2)$ and define its result as

$$H''_j(k1,k2) = (H'_j(k1,k2) - H'min(k1,k2))/(H'max(k1,k2) - H'min(k1,k2)) \tag{19}$$

where $H'min(k1,k2) = \min\{H'_j(k1,k2)\}$, and $H'max(k1,k2) = \max\{H'_j(k1,k2)\}$

6. Apply ED to $H''_j(k1,k2)$ and denote the resulting hologram $H'''_j(k1,k2)$. The direction of ED is shown in Fig. 6.

7. Compute the inverse discrete Fourier transform(IDFT) of $H'''_j(k1,k2)$, and define the reconstructed image corresponding to the image quadrant as $Xrecj(n1,n2)$.

8. Compute the total reconstruction error.

If some convergence criterion has been reached, stop the process, otherwise let

$$E_{i,j}(n1,n2) = X(n1,n2) - \lambda_{i,j} (Xrec1(n1,n2) + Xrec2(n1,n2)) \tag{20}$$

If this is the end of the ith sweep, let $i = i + 1$.

9. Go to step 1.

This procedure is shown in a flow chart in Fig. 7. A modification described below is also included in the flow chart to reduce the computation cost. The decimation process eventually discards the unused rows(i.e. set those rows to zero). Hence, it is only necessary to normalize the rows of interest. With H'max(k1,k2) and H'min(k1,k2) defined above, the normalization process normalizes the maximum and the minimum values of desired rows to 1 and 0. The other rows are not computed.

V. COMPUTER EXPERIMENTS

In the first experiment, a 64 x 64 girl image was used as the object image. The object superimposed with a random phase was placed inside a data field of 512 x 512 pixels and was centered at (32,-96). In the second experiment, a 64 x 32 binary image superimposed with a random phase was placed inside a data field of 256 x 256 and centered at (16,-64).

In the following, an iteration is defined to be a complete design of one hologram. In a two-hologram simulation, a sweep means two iterations. In Fig. 8, the: mean-square reconstruction error versus the ED coefficient o is shown. Curve A is the result of one iteration using one hologram, meaning ED alone. Curves B, C, D are calculated after 40 iterations or 20 sweeps in the case of two holograms. Curve B corresponds to the combination of the unmodified IIT and the ED techniques. In curves C and D, the modified IIT is used, with β = 1.0 and 1.5, respectively.

The result from curve A is not unexpected. Lower reconstruction error can be achieved using ED techniques with a larger ***o***. Curves of B,C,D show that further improvement can be obtained through iterative procedures. In all cases,the IIED technique had better results than those of unmodified IIT together with ED or IIT alone(i.e. $\omega = 0$). As an example, Fig. 9 shows that, except for a transient peak during the first or second sweep which was typical with the IIED technique, the reconstruction error steadily decreases without experiencing any stagnation problems. In the experiments, best results were obtained with β's between 1.0 and 2.0. Unlike the noniterative ED teclhniques which has best results with $\omega = 1$, values of ω close to 0.2 were found to be optimal for the IIED technique at $\beta = 1.5$. One possible explanation is that the stable range of ω with the IIED technique is smaller than those of noniterative ED techniques.

Fig. 10 shows the reconstructed images obtained with the noniterative ED and the IIED techniques with different error diffusion coefficients and iteration numbers.These images; are the desired pan of the total focal plane intensity image. The total focal plane intensity images are shown in Figs. 11 and 12, which were generated by hard-clipping all pixel values above the maximum value in the desired reconstructed image. This procedure was necessary for displaying the desired reconstruction image correctly with the image processing software used.

In Fig. 10, the original girl image is shown at the top, followed by two reconstruction images obtained with the ED technique. In cases C and D, from left to right and top to bottom, the images reconstructed using the IIED technique are with ω equal to 0.0 , 0.05 , 0.1 , 0.2 , 0.4 , 0.6 , 0.8 and 1.0, respectively. In case A,the larger ω was used, the more noise was driven away from the reconstructed image and a lower reconstruction error was obtained. However, due to a well-known fact that ED techniques suffer from low efficiencies, lower contrast images were obtained using larger ω's. Furthermore, even the best result among all noniterative cases(i.e. ED with $\omega = 1.0$), some portion of the image was corrupted with the diffusing noise. In contrast, iterative techniques such as IIT can reduce noise uniformly and obtain higher efficiency. Due to different effects of the two types of algorithms, the IIED technique sharing characteristics of both methods mentioned above turns out to be the best solution. In the computer experiments, the best result achieved using the ED technique and the IIED technique with ω = 0.2 and β = 1.5 are shown in Figs. 11 and 12, respectively. The corresponding IIED hologram is shown in Fig. 13.

Three types of noise can be described in terms of their locations and sources in Figs 11 and 12; first, the noise that is driven away from the reconsmcted image due to the ED process; second, next to the girl image, the noisy image with high intensity resulting from the interlacing technique; third, the noise left in the desired image region. In Fig. 11, since only the ED technique was used, most of the noise belongs to the first and the third kind. Due to the high intensity noise of the first kind, the image is reconstructed with lower contrast and lower efficiency. Also, some portion of the desired image is corrupted with noise of the third kind. In Fig. 12, the noise of the first kind with lower intensity, and the noise left in the desired image region is more uniform resulting in a higher contrast image with higher efficiency.

In the second part of computer simulations, a 64 * 32 binary image, which is part of an edge-enhanced image of the cross-section of a cat's brain, was used. The binary image is shown in Fig.14. Fig. 15 shows the mean-square error results, corresponding to Fig. 8 of the previous case. The results were very similar to those obtained with the girl image. In this case, the best performance was achieved using the IIED technique with ω = 0.1 and β = 1.5.

VI. CONCLUSIONS

The IIED technique consists of the combination of the ED technique and the modified IIT technique. Experimentally, with β= 1.5, ω =0.2 in the case of the girl image and ω = 0.1 in the case of the binary image, a fairly dramatic improvement in the quality of the reconstructed images resulted with the IIED technique. Unlike noniterative ED techniques, which use ω = 1.0 to drive the noise clouds away from the reconstructed image at the cost of lower efficiency, the IIED technique, using a small ω and β = 1.5, was shown both to preserve the characteristic of the ED technique separating noise clouds from the reconstructed image and to uniformly distribute the remaining reconstruction noise. These properties resulted in improved image contrast and higher diffraction efficiency.

In this article, the IIED technique was considered only for binary holograms, Its extension is possible to multi-level holograms with multi-level ED techniques[12] and phase holograms with complex ED techniques[7].

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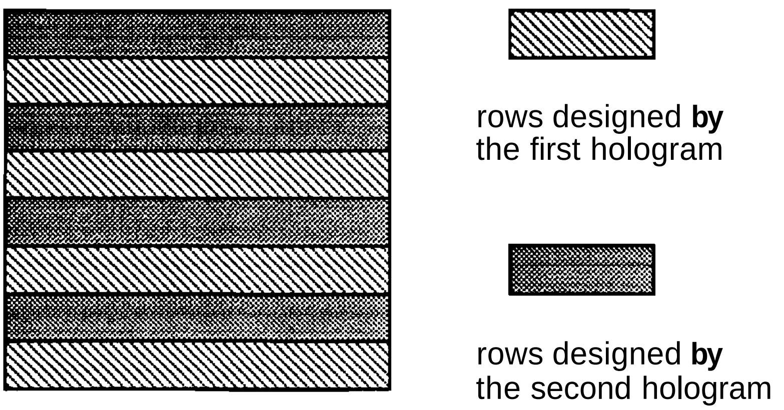


Fig. 1. The Geometry Used in Constructing Two Interlaced Holograms.

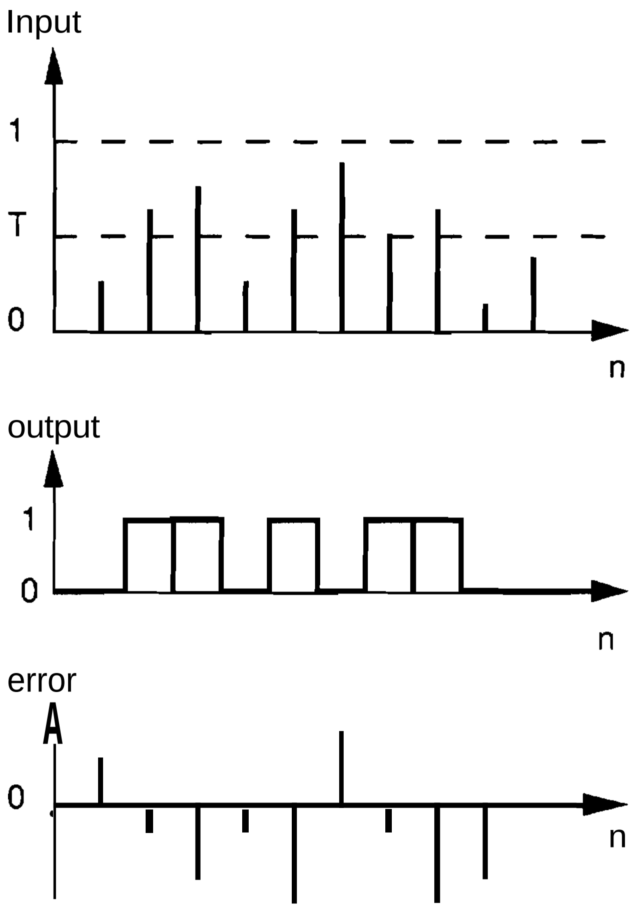


Fig. **2.** Error Diffusion.

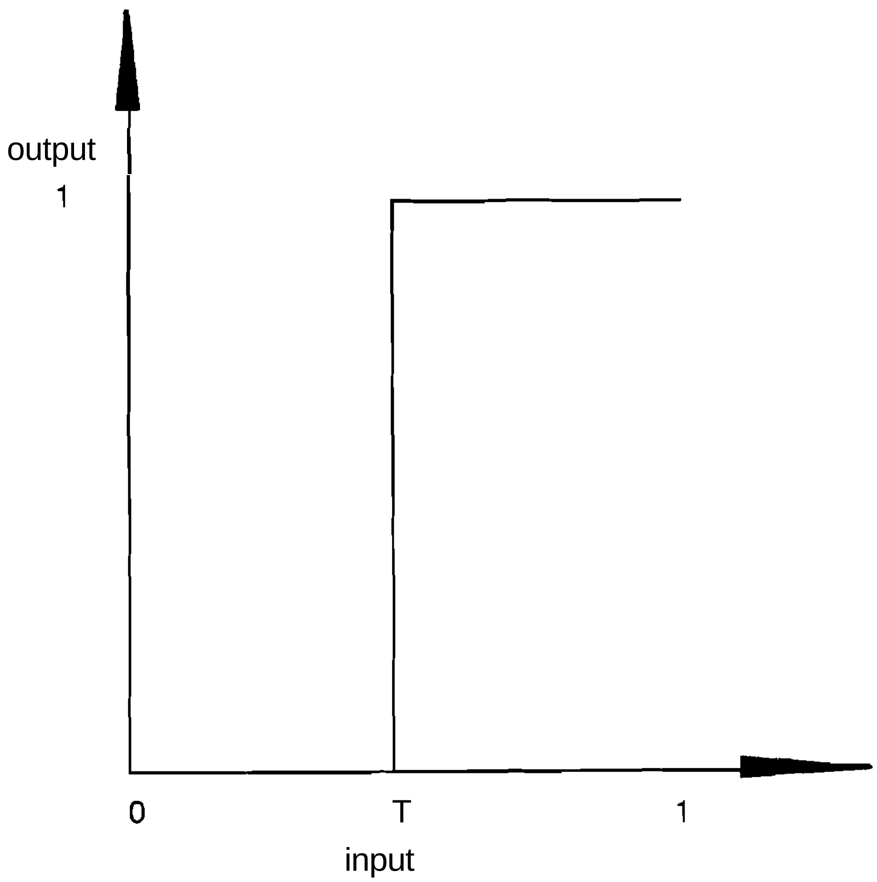


Fig. **3.** The Hard-Limiter Used in the ED Process.

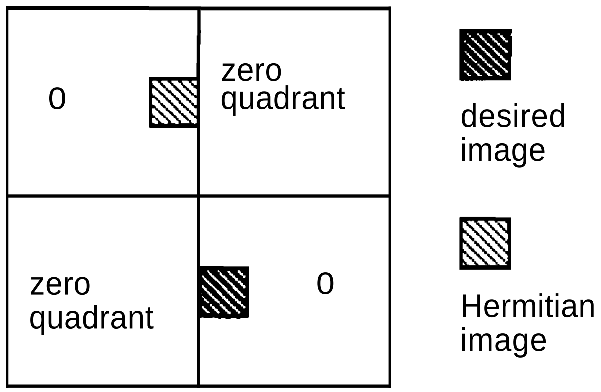


Fig. 4. The Placement of the Images and Zero Region!; in the ED and the IIED Techniques.

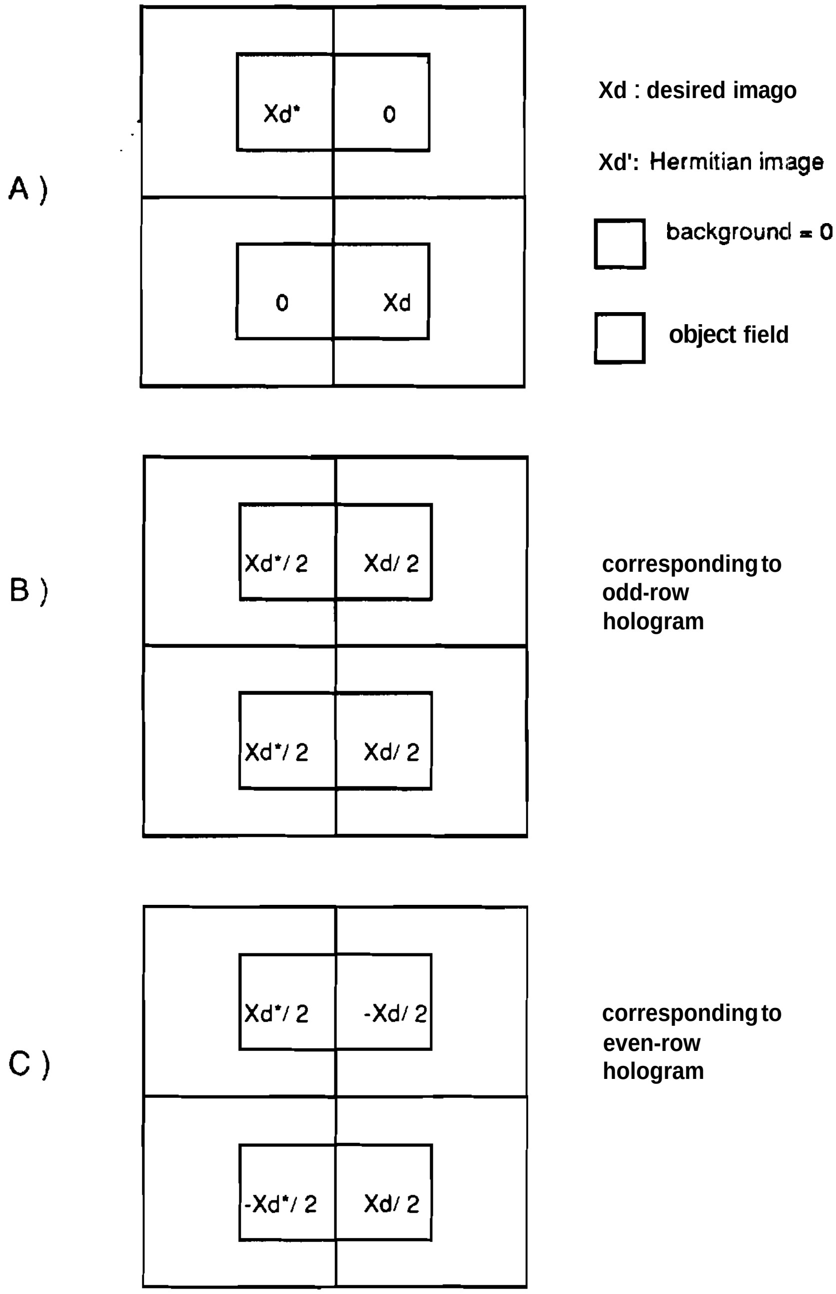


Fig. 5. Generation of Twin Images due to Decimation into Two Holograms: A) No Decimation B) Odd-Row Hologram Only C) Even-Row Hologram Only.

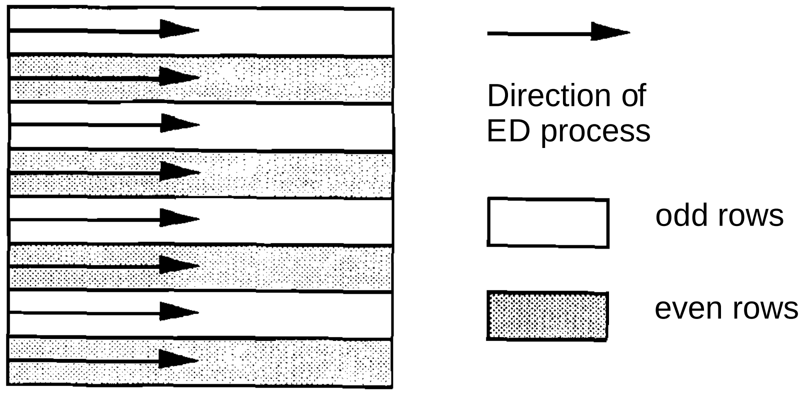


Fig. 6. The Direction of Error Diffusion in the Two Holograms of the IIED Technique.

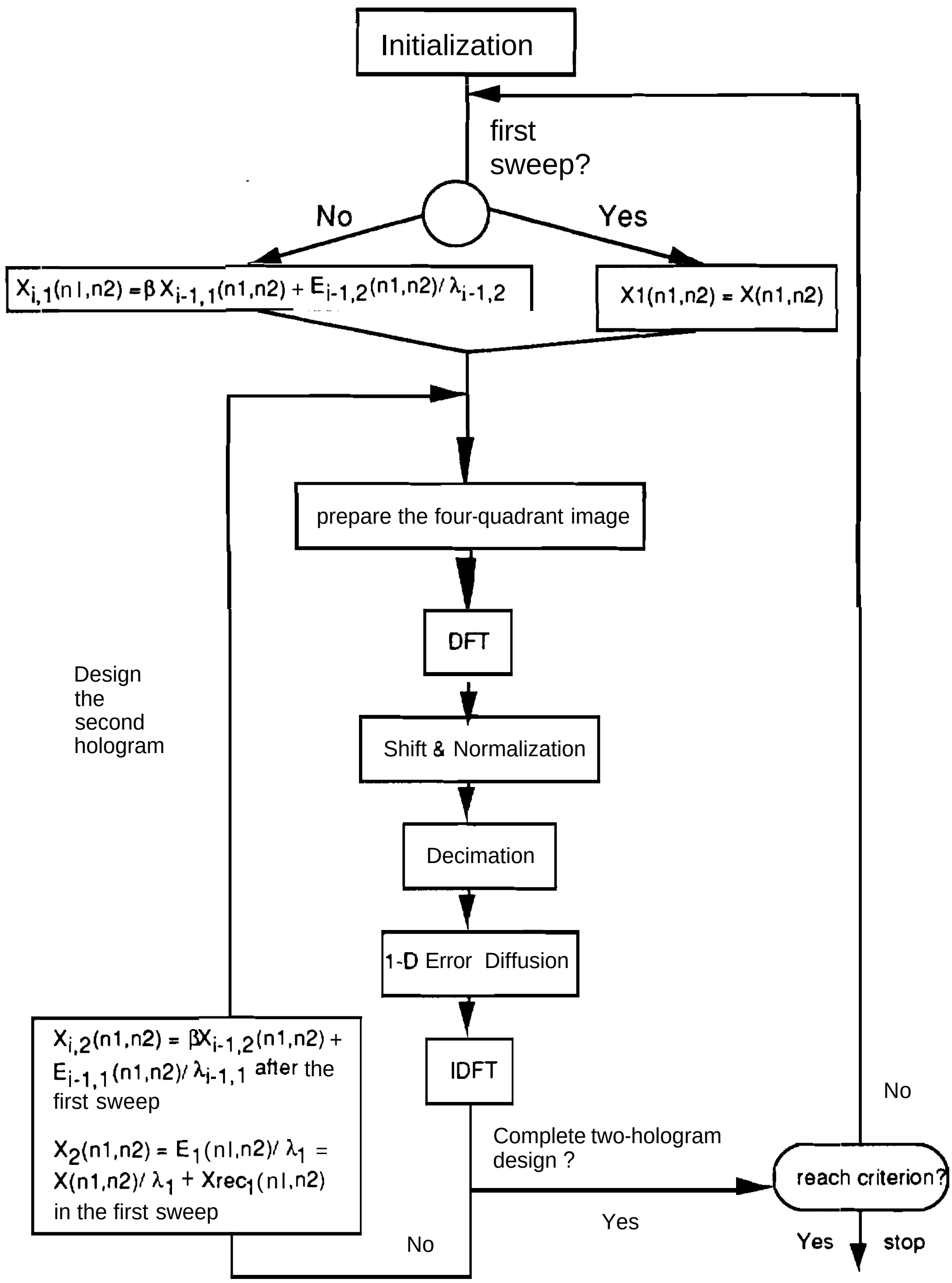
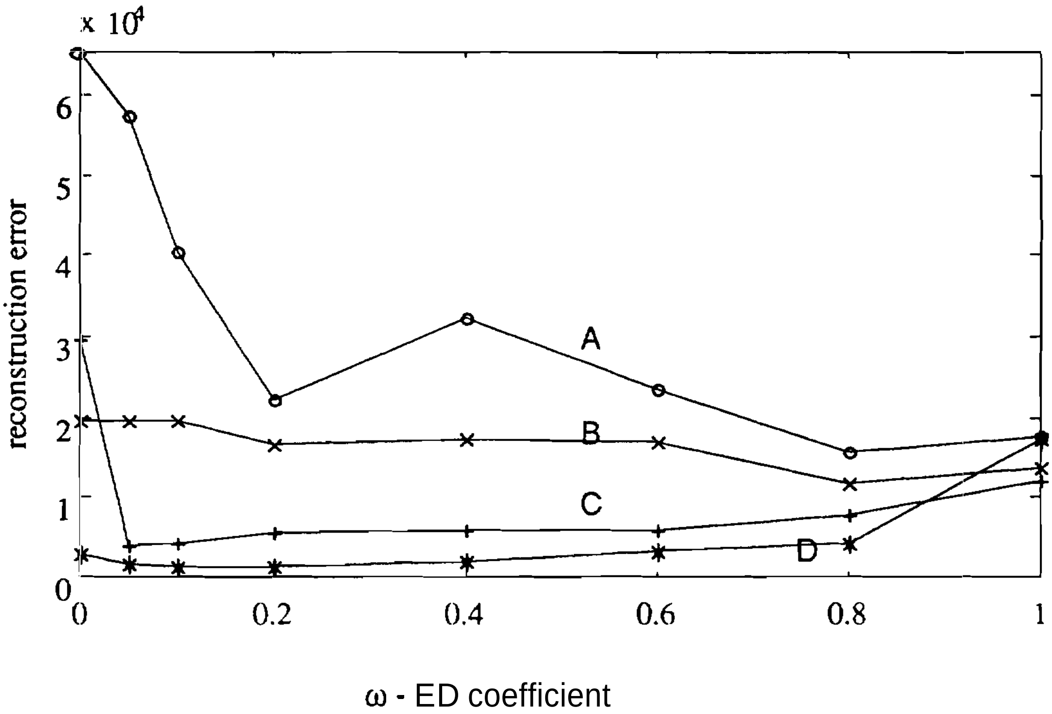


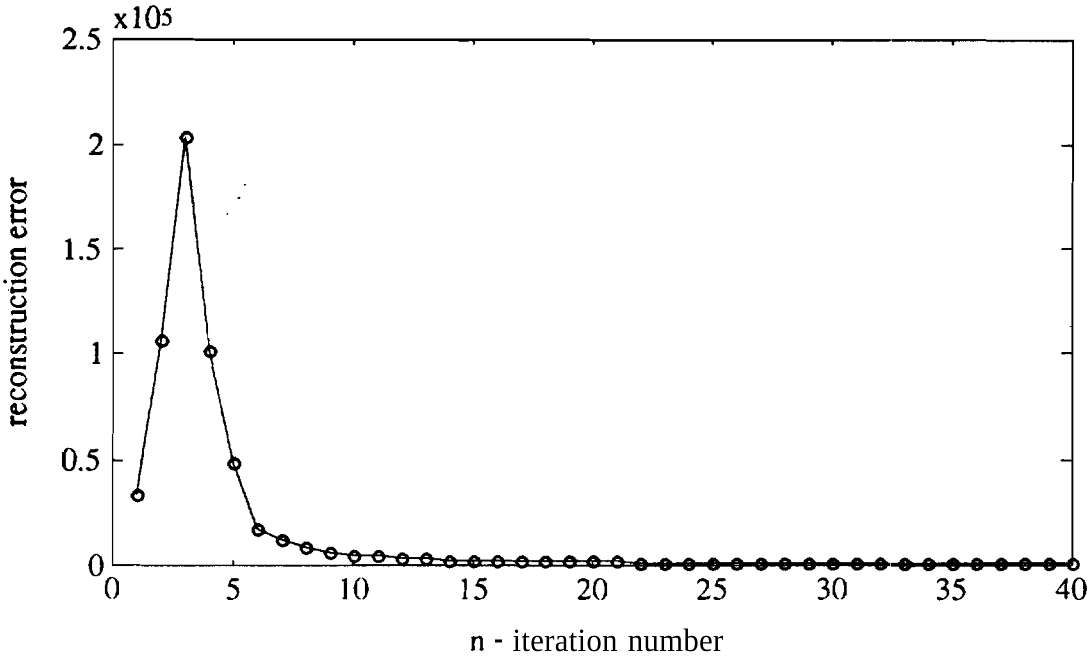
Fig. 7. The Flow Chart of the IIED Technique.



A - 1 iteration, 1 hologram, using ED
B - 20 sweeps, **2** holograms, using the unmodified TIT and ED
C - 20 sweeps, 2 holograms, using TIED with b = 1.0
D - 20 sweeps, 2 holograms, using IIED with b = 1.5

Fig. 8. The Reconstruction Error versus the ED Coefficient with the Three Techniques Used with the Girl Image.

A)



B)

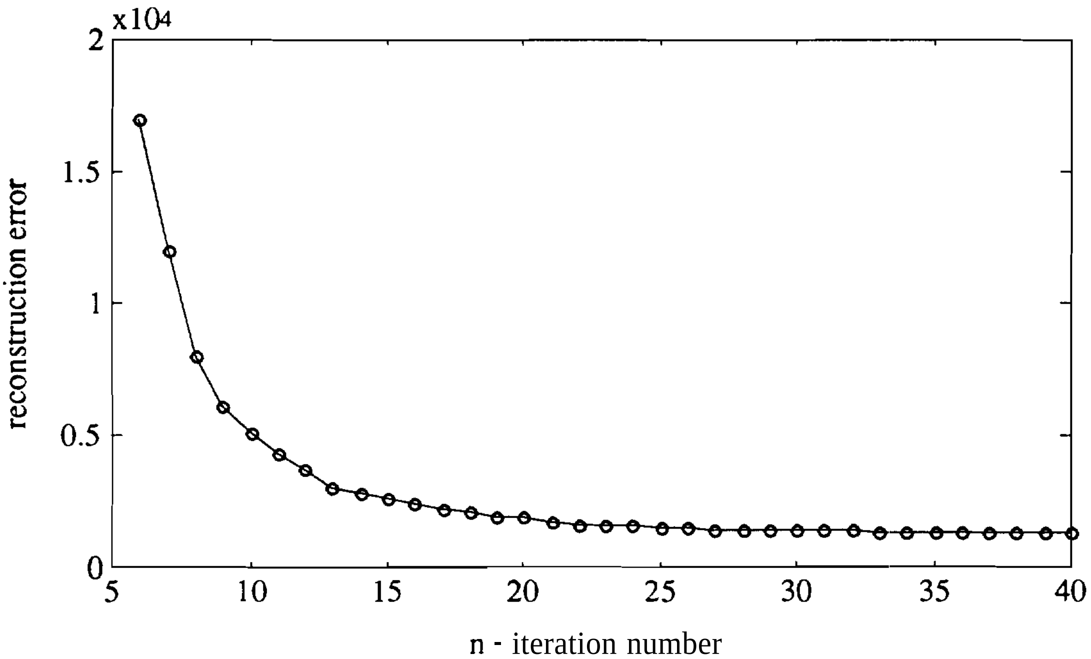


Fig. 9. A) The Reconstruction Error versus the Iteration Number in the IIED Technique B) The Enlarged Right-Hand Side of Fig. 9 .A.

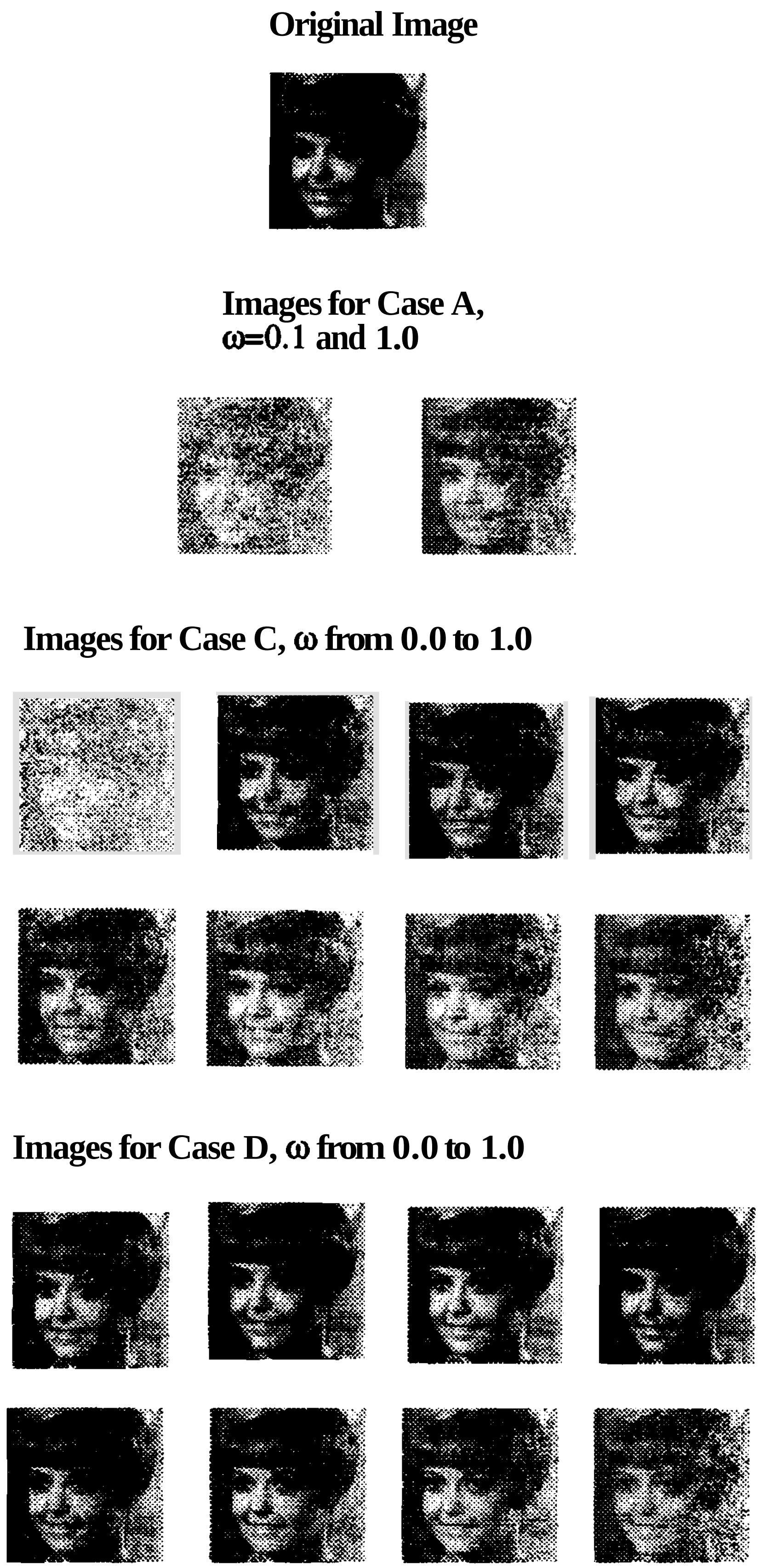


Fig. 10. The Reconstructed Images Obtained with the Three Techniques.

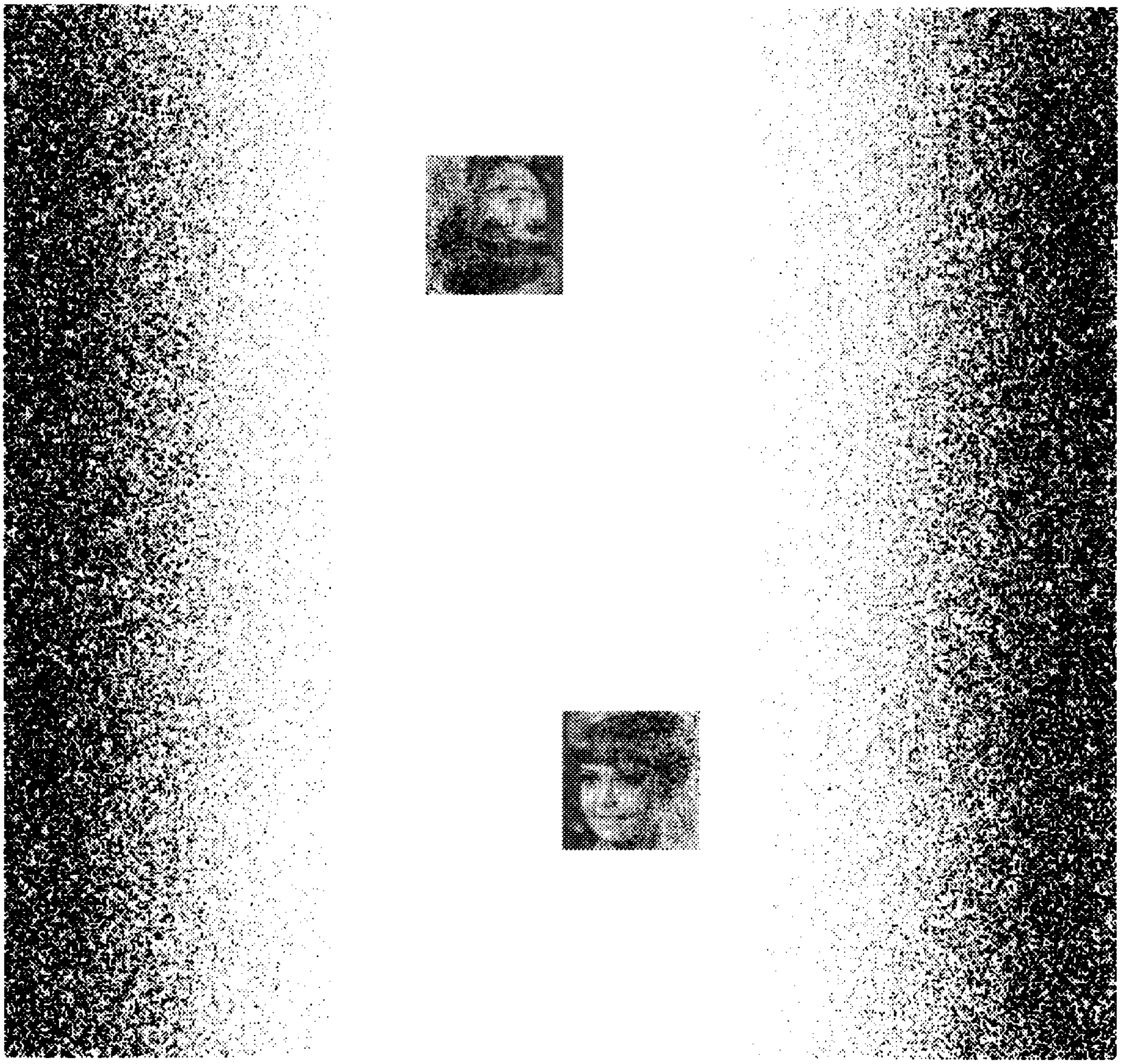


Fig. 11. The Total Reconstructed Image at the Focal Plane with the ED Technique.

Fig. 12. The Total Reconstructed Image at the Focal Plane with the IIED Technique.

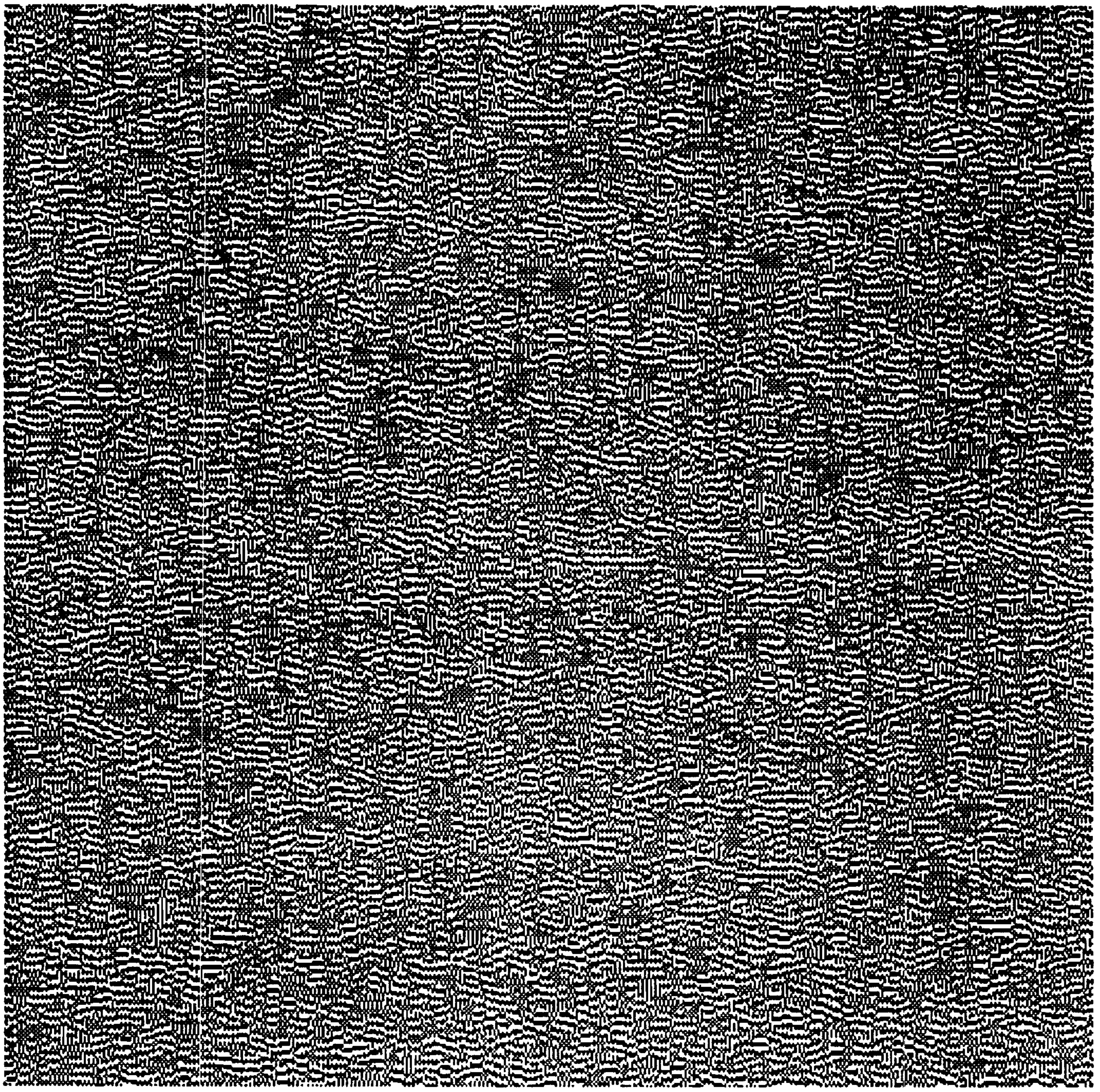
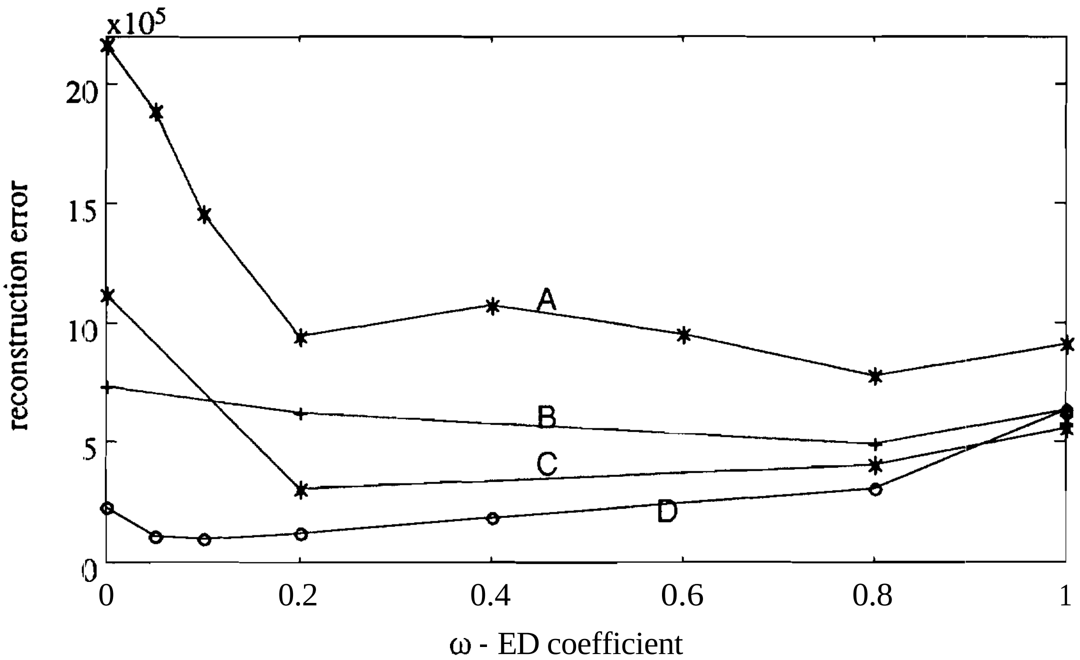


Fig. 13. The Hologram Generated by the IIED Technique for the Girl Image.



A - 1 iteration, 1 hologram, using ED
B - 10 sweeps, 2 holograms, using the unmodified IIT and ED
C - 10 sweeps, 2 holograms, using IIED with b = 1.0
D - 10 sweeps, 2 holograms, using IIED with b = 1.5

Fig. 14. The Reconstruction Error versus the ED Coefficient ω in the Three 'Techniques Used with The Binary Image.

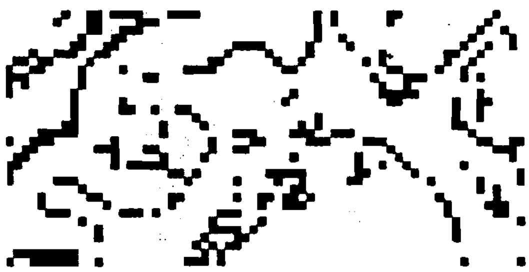


Fig. 15. The Binary Image Used in the Second Set of Computer Experiment.